terference effects as a cause for the stepwise variation of initial instability frequency with jet exit velocity.

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Dynamic Buckling of Orthotropic Spherical Caps Supported by Elastic Media

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Introduction

THE buckling of isotropic shallow spherical shells has been the subject of extensive studies.[1-13] There are few investigations[14,15] dealing with the dynamic buckling of orthotropic shallow spherical shells. In these studies the effect of interaction with the supporting elastic media on the dynamic buckling of spherical caps has not been considered. The dynamic response of isotropic shallow spherical shells supported on a Winkler-Pasternak elastic foundation has been investigated recently.[16] The studies of the dynamic buckling of orthotropic shallow spherical shells with or without holes interacting with supporting elastic media are not available in the literature and are dealt with for the first time in this paper.

Von Kármán-Marguerre-type governing nonlinear partial differential equations for the orthotropic spherical caps on a Winkler-Pasternak[17] elastic foundation are employed; they are linearized using the quadratic extrapolation technique and solved iteratively using the Chebyshev series and Houbolt schemes for the space and time domains respectively.[14,16] The conditions of finiteness at the center for the full cap and free inner edge for the annular cap are assumed. Two criteria, namely, a sudden jump[3] in the average deflection response and the point of inflection[8] in the load vs maximum average deflection curve, are used to estimate the dynamic buckling loads. The influence of foundation stiffness, orthotropy, and annular ratio on the dynamic buckling load of immovably clamped and simply supported shallow spherical caps is investigated, and typical results are presented.

Mathematical Analysis

The geometry of a shallow spherical shell on a Winkler-Pasternak elastic foundation is shown in Fig. 1, in which a and b are outer and inner radii, h is the thickness of the shell, K and G are foundation stiffnesses, and w and u are normal and meridional deflections, respectively. Considering the cylindrically orthotropic material whose elastic axis of orthotropy coincides with the axis of symmetry of the shallow spherical shell of constant curvature K^*, the governing partial differential equation of motion and compatibility condition in terms of dimensionless normal deflection $\bar{w}$ and stress function $\bar{\psi}$ for a thin shallow spherical shell supported by a Winkler-Pasternak elastic foundation and undergoing

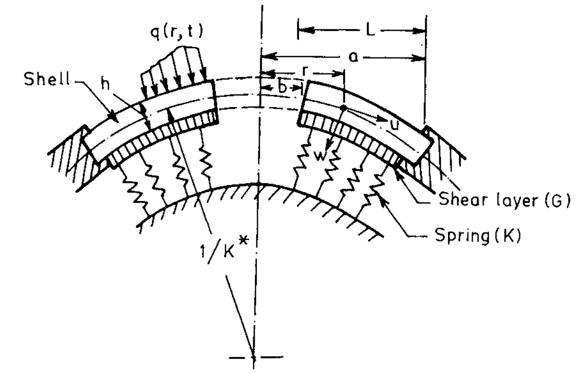

Fig. 1 Geometry of a shallow annular spherical cap on elastic foundation.

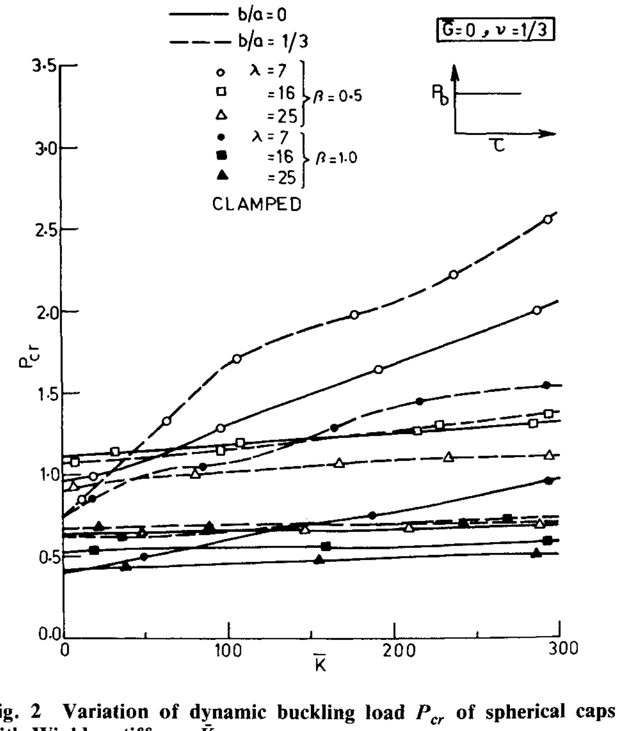

Fig. 2 Variation of dynamic buckling load P_{cr} of spherical caps with Winkler stiffness $\bar{K}$.

Received Dec. 20, 1985; revision received July 17, 1986.

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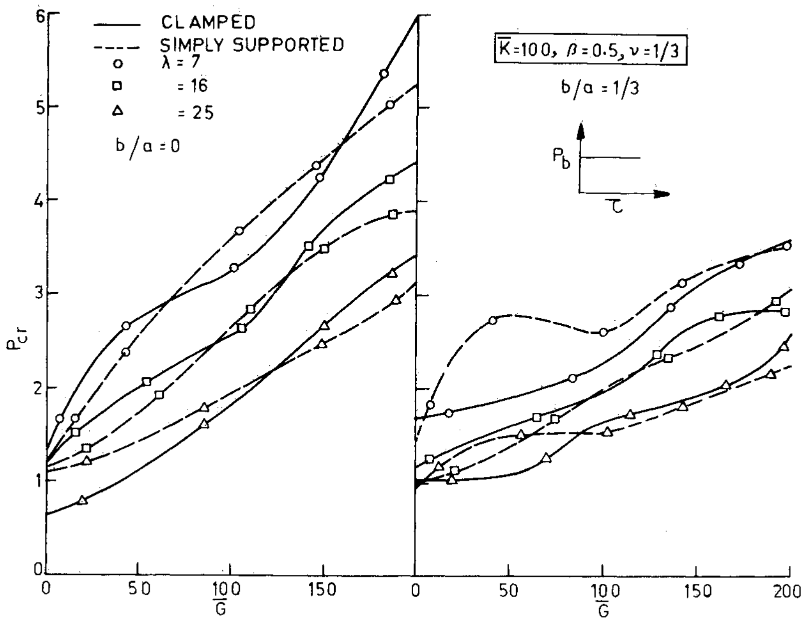

Fig. 3 Variation of dynamic buckling load P_{cr} of spherical caps with Pasternak shear stiffness $\bar{G}$.

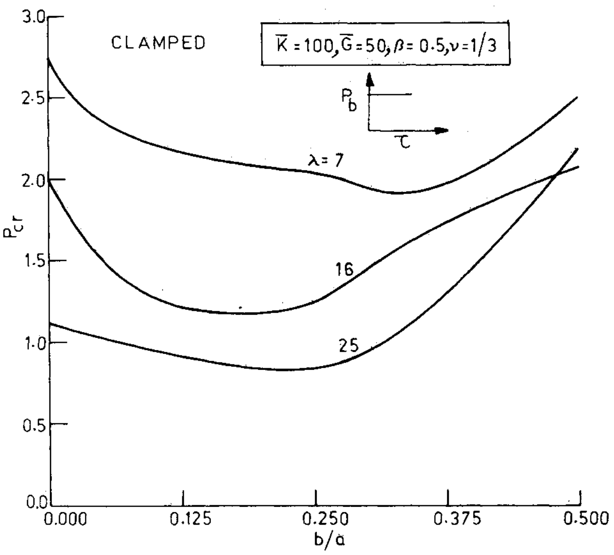

Fig. 4 Variation of dynamic buckling load P_{cr} of spherical caps with annular ratio b/a.

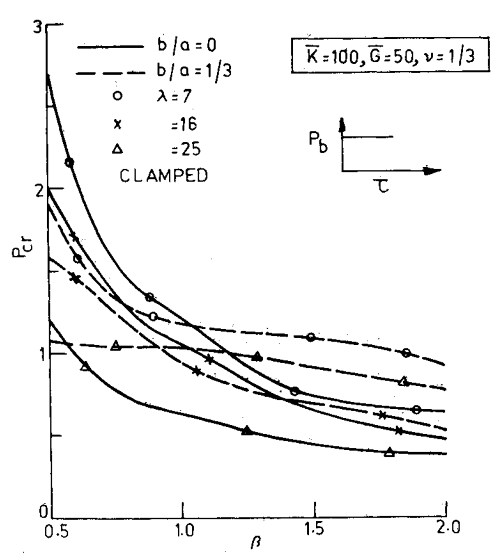

Fig. 5 Variation of dynamic buckling load P_{cr} of spherical caps with orthotropy β.

moderately large dynamic deformation can be expressed as

$$(\rho+\xi)^2\bar{w}''' + (\rho+\xi)\bar{w}'' - \beta\bar{w}' - 12\left(\frac{L}{h}\right)^2(\rho+\xi)(\bar{\psi}\cdot\bar{w}')$$
$$-12\lambda\left(\frac{L}{h}\right)(\rho+\xi)^2\bar{\psi} + (\rho+\xi)\int_0^\rho [(\rho+\xi)(\bar{K}\bar{w}-\bar{G}\bar{w}'')$$
$$-\bar{G}\bar{w}']\,\mathrm{d}\rho - (\rho+\xi)\int_0^\rho (\rho+\xi)\left\{24\left(\frac{L}{a}\right)^3\left(\frac{h}{a}\right)\right.$$
$$\left.\times\frac{(\beta-\nu^2)\lambda^2}{[3(1-\nu^2)]^{1/2}}P_b - \ddot{\bar{w}}\right\}\mathrm{d}\rho = 0 \qquad (1)$$

$$(\rho+\xi)^2\bar{\psi}'' + (\rho+\xi)\bar{\psi}' - \beta\bar{\psi} + 0.5(\beta-\nu^2)(\rho+\xi)$$
$$\times\left[(\bar{w}')^2 + 2\lambda\left(\frac{h}{L}\right)(\rho+\xi)\bar{w}'\right] = 0 \qquad (2)$$

where superscripts ( )′ and (·) are derivatives with respect to ρ and τ respectively. The nondimensional and physical quantities are related through the following relations:

$$L = a-b,\quad \xi = \frac{b}{a-b},\quad \rho = \frac{r-b}{L},\quad \bar{w} = \frac{w}{L}$$

$$\bar{\psi} = a_{11}\frac{(\beta-\nu^2)}{hL}\psi,\quad \bar{K} = \frac{KL^4}{D},\quad \bar{G} = \frac{GL^2}{D}$$

$$P = \frac{qa^3}{D},\quad \lambda = \frac{K^*L^2}{h},\quad \tau = \left[\frac{D}{\gamma hL^4}\right]^{1/2} t$$

$$P_{cl} = \frac{2\lambda^2 Eh^4}{[3(1-\nu^2)]^{1/2}a^4},\quad \nu = -\frac{a_{11}}{a_{22}}$$

$$P_b = \frac{q}{P_{cl}} = \frac{qa^4[3(1-\nu^2)]^{1/2}}{2\lambda^2 Eh^4} = P\frac{[3(1-\nu^2)]^{1/2}}{24\lambda^2(\beta-\nu^2)}\left(\frac{a}{h}\right)$$

$$\beta = \frac{a_{22}}{a_{11}},\quad E = \frac{1}{a_{11}},\quad D = \frac{Eh^3}{12(\beta-\nu^2)}$$

where a_{11}, a_{22}, and a_{12} are the elastic constants.

The boundary conditions are

At $\rho=0$:

$$\bar{w}'=\bar{\psi}=0 \quad \text{for full shell} \tag{3}$$

$$(\rho+\xi)\bar{w}''+\nu\bar{w}'=\bar{\psi}=0 \quad \text{for annular shell} \tag{4}$$

At $\rho=1$:

$$\bar{w}=\bar{w}'=(\rho+\xi)\bar{\psi}'-\nu\bar{\psi}=0 \quad \text{for clamped edge} \tag{5}$$

$$\bar{w}=(\rho+\xi)\bar{w}''+\nu\bar{w}'=(\rho+\xi)\bar{\psi}'-\nu\bar{\psi}=0 \quad \text{for simply supported edge} \tag{6}$$

The initial conditions are assumed that at $\tau=0$, $\bar{w}(\rho,0)=\dot{\bar{w}}(\rho,0)=0$. The nondimensional average deflection (characteristic deflection) is defined as

$$\bar{W}=\left(\frac{L}{a}\right)^2\left(\frac{L}{h}\right)[48(1-\nu^2)]^{1/2}\int_0^\rho(\rho+\xi)\bar{w}\mathrm{d}\rho \tag{7}$$

Equations (1) and (2), along with the appropriate boundary and initial conditions, are solved iteratively, employing the Chebyshev series and Houbolt time marching techniques.[14,16] The iterations are continued until $\bar{w}(0,\tau)$, $\bar{\psi}'(0,\tau)$, and $\bar{\psi}'(1,\tau)$ satisfy a relative convergence criteria within 0.1% accuracy.

Results and Discussion

The space and time wise convergence studies have revealed that 15 terms in the Chebyshev series for functions $\bar{w}$ and $\bar{\psi}$ and time step $\Delta\tau=0.002$ yield quite accurate results for the range of parameters λ, β, $\bar{K}$, $\bar{G}$, and b/a considered in the present study. Adopting the dynamic buckling criteria suggested by Budiansky and Roth[3] and Stephens and Fulton,[8] the nonlinear axisymmetric dynamic buckling of cylindrically orthotropic shallow spherical caps with and without holes has been studied under uniformly distributed step function loading. The influence of Winkler stiffness $\bar{K}$, Pasternak shear stiffness $\bar{G}$, orthotropy β, and annular ratio b/a on the dynamic buckling load of shallow spherical caps is investigated for several values of shell geometric parameter λ. Results have been compared with available results[5,7,8,11,14,15] for isotroic and orthotropic shallow spherical shells, and it is observed that the present results are in close agreement with the results reported by some of the researchers and fall within close range of the results reported by the others. For the sake of brevity, these are not presented here.

The variation of the dynamic buckling load P_{cr} with the Winkler stiffness parameter $\bar{K}$ for the clamped orthotropic annular spherical shells is depicted in Fig. 2 for $b/a=0$ and $1/3$, $\beta=0.5$ and 1.0, and $\lambda=7$, 16, and 25. It can be noted that the higher the value of $\bar{K}$ the higher the buckling load. The increase in the value of P_{cr} is more for the orthotropic shells ($\beta\leqq1.0$) as compared to the corresponding isotropic shells ($\beta=1.0$). It can also be seen that there is a little increase in the buckling loads of spherical shells for $\lambda=16$ and 25 with an increase of $\bar{K}$, but there is a significant increase in the snap-through loads of spherical shells for $\lambda=7$.

The variation of the dynamic buckling load P_{cr} with the Pasternak shear stiffness $\bar{G}$ for the clamped and simply orthotropic annular spherical caps is shown in Fig. 3 for $b/a=0$ and $1/3$, $\beta=0.5$, $\bar{K}=100$, and $\lambda=7$, 16, and 25. It can be seen that the buckling load P_{cr} increases with an increase in the value of $\bar{G}$ for both the clamped and simply supported spherical shells and that P_{cr} for $\lambda=7$ is greater than for $\lambda=16$ and 25 for all values of $\bar{G}$ considered. In the case of annular shells, it is interesting to note that for $\lambda=7$, the buckling load P_{cr} is more for the simply supported shell than for the clamped shell for $5\leqq\bar{G}\leqq185$. It is noted that an increase in the buckling load with Pasternak shear stiffness $\bar{G}$ is significant for all values of the shell geometric parameter λ considered in the present study.

The effect of hole size (b/a) on the dynamic buckling load of orthotropic spherical caps supported on a Winkler-Pasternak elastic foundation has been investigated, and the results are shown in Fig. 4 for $\beta=0.5$, $\bar{K}=100$, $\bar{G}=50$, and $\lambda=7$, 16, and 25. It can be observed that the buckling load P_{cr} decreases initially and increases finally with an increase of the annular ratio b/a.

The variation of the dynamic buckling load P_{cr} with the orthotropic parameter β for the clamped spherical caps with and without holes ($b/a=0$ and $1/3$) on a Winkler-Pasternak elastic foundation ($\bar{K}=100$, $\bar{G}=50$) is plotted in Fig. 5 for shell geometric parameter $\lambda=7$, 16, and 25. It is clear from the results that the buckling load decreases with an increase of the orthotropic parameter. It can be observed that the buckling loads for orthotropic shells for $\beta<1$ are higher than the buckling load of isotropic shells ($\beta=1.0$). The rate of change of P_{cr} with orthotropy is less for $\beta>1$ than for $\beta<1$.

Conclusions

The dynamic buckling of clamped and simply supported cylindrically orthotropic shallow spherical shells with and without holes supported by a Winkler-Pasternak elastic foundation and subjected to step loading has been studied. The influence of foundation stiffness, hole size, and orthotropy on the dynamic buckling load has been investigated. It can be concluded from the study that the buckling loads are higher for larger values of foundation stiffness, lower value of orthotropy, and larger annular ratio. Also, the buckling loads of spherical caps having a very small ratio of inner to outer radii are lesser than the buckling loads of spherical caps without holes.

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